**Essential Question** How can you solve an equation that has variables on both sides?

### 1 ACTIVITY: Using a Table, Graph, and Algebra

**Work with a partner. You have an email pal in Antarctica. Your email pal tells you the temperature in McMurdo. You ask whether he gave the temperature in Celsius or Fahrenheit. He says "It's the same on both scales." What is the temperature?**

a. **TABLE** Use "Guess, Check, and Revise" with a table to find the only temperature that is the same on both scales.

| F | | | | | | | |
|---|---|---|---|---|---|---|---|
| C | | | | | | | |

b. **GRAPH** Draw the line given by $C = F$ in the coordinate plane. Locate the point at which the graph of $C = F$ intersects the graph of
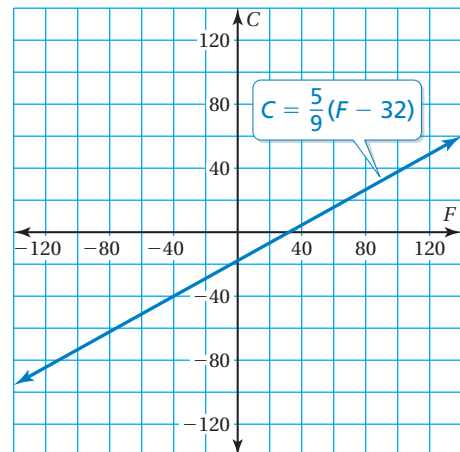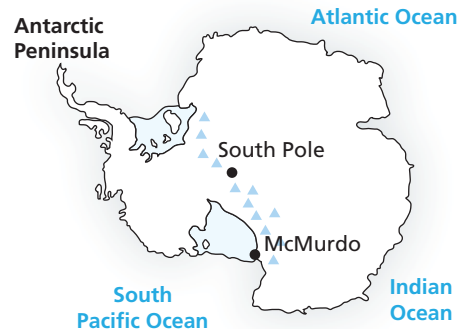
$$C = \frac{5}{9}(F - 32).$$

c. **ALGEBRA** Let $x$ be the temperature that is the same on both scales. Substitute $x$ for $C$ and $F$ in the equation

$$C = \frac{5}{9}(F - 32).$$

Then solve for $x$.

d. Compare your solutions from parts (a)–(c). Did you get the same solution with each method? Which method do you prefer? Why?

**Antarctic Peninsula**

**Atlantic Ocean**

South Pole

McMurdo

**South Pacific Ocean**

**Indian Ocean**

$$C = \frac{5}{9}(F - 32)$$

## GAME: Race to the South Pole

**Play with a partner.**

- Write each expression on a scrap of brown or blue paper. Place the brown pieces of paper in one bag and the blue pieces of paper in another bag.
- Draw an expression from each bag and set them equal to each other.
- If you can solve the equation, you move one space. If you cannot solve the equation, your partner gets a chance to solve it and move one space.
- Put the expressions back into their bags.
- Take turns. The first person to reach the South Pole wins.

| Brown Papers | Blue Papers |
|---|---|
| $x$ | $2x$ |
| $x + 1$ | $2x + 4$ |
| $x - 1$ | $-2x$ |
| $x + 2$ | $-2x + 4$ |
| $x - 2$ | $3x$ |
| $x + 3$ | $3x + 6$ |
| $x - 3$ | $-3x$ |
| | $-3x + 6$ |

$$x + 1 \; = \; 3x$$



# What Is Your Answer?

3. **IN YOUR OWN WORDS**  How can you solve an equation that has variables on both sides? Give an example and solve it.

**Practice** ▶  Use what you learned about solving equations with variables on both sides to complete Exercises 4–9 on page A20.

To solve an equation with variables on both sides, collect the variable terms on one side and the constant terms on the other side.

### EXAMPLE 1 Solving an Equation with Variables on Both Sides

**Solve $-4y = 9 - y$. Check your solution.**

| | |
|---|---|
| $-4y = 9 - y$ | Write the equation. |
| $\underline{+y \qquad +y}$ | Add $y$ to each side. |
| $-3y = 9$ | Simplify. |
| $\dfrac{-3y}{-3} = \dfrac{9}{-3}$ | Divide each side by $-3$. |
| $y = -3$ | Simplify. |

Collect the variable terms.

**Check**

$$-4y = 9 - y$$
$$-4(-3) \stackrel{?}{=} 9 - (-3)$$
$$12 = 12 \checkmark$$

### EXAMPLE 2 Using the Distributive Property to Solve an Equation

**Solve $-4\left(\dfrac{7}{2}x + 1\right) = 2(x - 4)$.**

| | |
|---|---|
| $-4\left(\dfrac{7}{2}x + 1\right) = 2(x - 4)$ | Write the equation. |
| $-14x - 4 = 2x - 8$ | Use Distributive Property. |
| $\underline{-2x \qquad\quad -2x}$ | Subtract $2x$ from each side. |
| $-16x - 4 = -8$ | Simplify. |
| $\underline{+4 \qquad +4}$ | Add 4 to each side. |
| $-16x = -4$ | Simplify. |
| $\dfrac{-16x}{-16} = \dfrac{-4}{-16}$ | Divide each side by $-16$. |
| $x = \dfrac{1}{4}$ | Simplify. |

Collect the variable terms.

Collect the constant terms.

### On Your Own

Now You're Ready
Exercises 4–22

**Solve the equation. Check your solution.**

1. $-5x = x + 12$
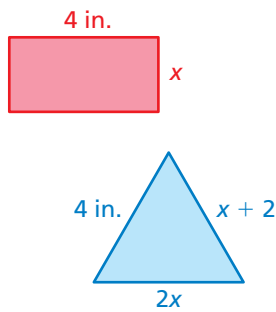
2. $15 - 4z = z - 8$

3. $6x = 2(7x + 3)$

4. $-3(1 - x) = \dfrac{1}{2}(2x + 1)$

## EXAMPLE 3  Standardized Test Practice

**The polygons have the same perimeter. What is the area of the rectangle?**

4 in.

$x$

(A) 12 in.     (B) 8 in.$^2$     (C) 12 in.$^2$     (D) 24 in.$^2$

The perimeter of the rectangle is $x + x + 4 + 4 = 2x + 8$.

The perimeter of the triangle is $4 + 2x + x + 2 = 3x + 6$.

4 in.   $x + 2$

$2x$

| Perimeter of rectangle | Perimeter of triangle |

$$2x + 8 = 3x + 6 \qquad \text{Write an equation.}$$
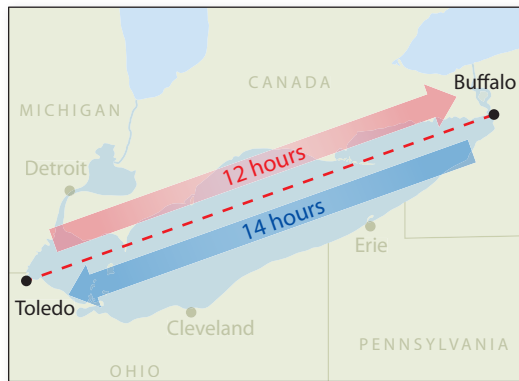$$8 = x + 6 \qquad \text{Subtract } 2x \text{ from each side.}$$
$$2 = x \qquad \text{Subtract 6 from each side.}$$

∴ So, the area of the rectangle is $2(4) = 8$ square inches. The correct answer is (B).

## EXAMPLE 4  Real-Life Application

**A boat travels $x$ miles per hour from Toledo to Buffalo. The boat travels 3 miles per hour slower on the return trip. What is the distance from Toledo to Buffalo?**

CANADA    Buffalo

MICHIGAN

Detroit

12 hours

14 hours

Erie

Toledo

Cleveland

PENNSYLVANIA

OHIO

The speed, or rate, of the boat on the return trip is $(x − 3)$ miles per hour.

| Distance from Toledo to Buffalo | Distance from Buffalo to Toledo |

$$12x = 14(x − 3) \qquad \text{Write an equation.}$$
$$12x = 14x − 42 \qquad \text{Use Distributive Property.}$$
$$−2x = −42 \qquad \text{Subtract } 14x \text{ from each side.}$$
$$x = 21 \qquad \text{Divide each side by } −2.$$

The boat travels 21 miles per hour from Toledo to Buffalo in 12 hours. So, the boat travels $12(21) = 252$ miles.

∴ The distance from Toledo to Buffalo is 252 miles.

### On Your Own

Now You're Ready
Exercises 25–27

**5.** In Example 3, what is the perimeter of the triangle?

**6.** In Example 4, what is the speed of the boat on the return trip?

## ✔ Vocabulary and Concept Check

1. **REASONING** Describe the steps you would use to solve $5x = 3x + 4$.

2. **OPEN-ENDED** Write an equation with variables on both sides that has a solution of $-1$.

3. **WRITING** To solve $-3x + 4 = -2x$, is it easier to add $3x$ to each side or add $2x$ to each side? Explain.

## Practice and Problem Solving

**Solve the equation. Check your solution.**

**4.** $2x = -x - 6$

**5.** $-9 + c = 4c$

**6.** $-3y = 7y$

**7.** $-5a = 2 - a$

**8.** $6w - 5 = 8w$

**9.** $3z = 14 + 10z$

**10.** $4f + 8 = 9f - 12$

**11.** $3p - 11 = 5p + 6$

**12.** $-7k + 10 = 9k + 18$

**13.** $-2 + 2d = 6d + 6$

**14.** $8b - 7 = -11 + 3b$

**15.** $12h - 7 = 6h + 8$

16. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗
$$3x - 7 = -2x + 8$$
$$3x + (-2x) = 8 + 7$$
$$x = 15$$

**Solve the equation. Check your solution.**

**17.** $2k - 8 = 4(k + 1)$

**18.** $3(g - 3) = 2(6 - 2g)$

**19.** $-5(f + 7) = 3(3f - 1)$

**20.** $9.2 - 4w = -2(3w + 5)$

**21.** $2.5(3b - 4) = 3.5b - 6b$

**22.** $6(1.5h - 1) = 5(2.2h + 3)$

23. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗
$$2(v - 5) = -(3v + 5)$$
$$2v - 10 = -3v + 5$$
$$5v = 15$$
$$v = 3$$

24. **MUSIC LESSONS** It costs $50 to be a member of a music club. A member of the club pays $10 per music lesson. A nonmember pays $20 per music lesson. How many music lessons must a member and a nonmember take so that the cost for each is the same?

**In a regular polygon each side has the same length. Find the perimeter of the regular polygon.**

**③** **25.**



$3x - 3$  $2x + 1$

**26.**



$x - 8$

$\frac{1}{3}x$

**27.**



$7 - 2x$

$4x - 5$
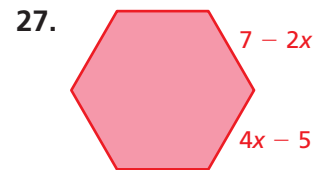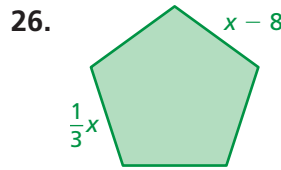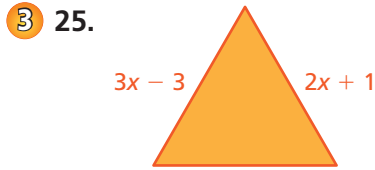
**Write an equation that describes the situation. Then solve the equation.**

**28.** Two more than the product of a number $n$ and 3 is equal to the product of the number $n$ and 4.

**29.** Three less than 5 times a number $n$ is equal to 8 more than negative 6 times the number $n$.

**30. SAVINGS** The table shows the beginning balance in each savings account. You save $10 per week and your friend saves $17 per week. How many weeks will it take for you and your friend to have the same balance?

| Savings Account Balance | |
|:---:|:---:|
| **You** | **Your Friend** |
| $126 | $84 |

**31. FLIGHT** A nonstop flight from Billings to Minneapolis takes $x$ hours. The return flight takes 30 minutes longer. What is the distance from Billings to Minneapolis?
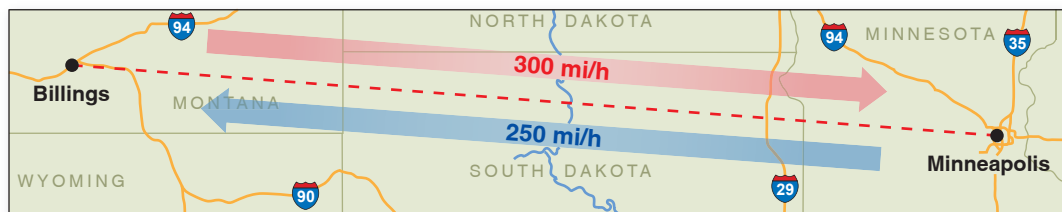


**32. PLAYGROUND** There are 87 children at a playground. There are 23 more boys than girls. How many of each are at the playground?

**33. GEOMETRY** $\triangle ABC$ is similar to $\triangle DEF$. The ratio of $AC$ to $DF$ is $3:5$. What is the perimeter of each triangle?
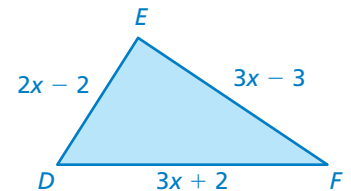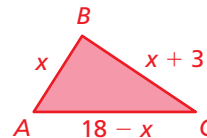


**34.** **Critical Thinking** Is it possible for an equation with variables on both sides to have no solution? If so, give an example.

**🄰** *Fair Game Review* **What you learned in previous grades & lessons**

**Graph the function.** *(Skills Review Handbook)*

**35.** $y = 2x - 1$

**36.** $y = 4 - x$

**37.** $y = 0.5x + 2$

**38. MULTIPLE CHOICE** What is the theoretical probability of rolling an even number on a number cube? *(Section 9.2)*

Ⓐ $\frac{1}{6}$   Ⓑ $\frac{1}{3}$   Ⓒ $\frac{1}{2}$   Ⓓ $\frac{2}{3}$